Mathematical Logic.

unit-I

Definition :- A Proposition (statement). :- It is well defined argument, which is either true or false but not both. The truth or falsity of a statement is called its truth value.

Example :- 1) Tirupati is in Andhra Pradesh.

(2) $8+3=11$

(3) Close the door

(4) where are you going ?

(5) Put the home work on the black board.

Note 1 :- (1) & (2) are statements.
(3) & (4) & (5) are not statements. Since neither true nor false

Note 2 : statements are denoted by P, Q, R --- (or) P, q, r, ---- (or) A, B, C, --- (or) P_1, P_2, P_3 ---- while declarative sentences are of the second type are obtained from the Primitive ones by using certain symbols, called Connectives

Note 3 :- T, F will be used for true, false respectively.
Some use 1, 0 for

Definition: Logic :- Logic is a method concerned wi
all kinds of reasoning where there be legal ~~sentences~~,
arguments (or) Mathematical Proof (or) Conclusions
in a scientific theory based upon a set of
hypothesis.

Statements are of two types

(1) Primary (or) primitive (or) atomic statement.

(2) Compound statement.

Statements that are considered initially in an object language are called atomic (or) Primary Statement.

New statements can be formed from atomic statements through the use of sentential connectives. The resulting statements are called molecular or compound statements.

Eg:- Let p be the statement - The Sun is rising in the East

Let q be the statement - It is Snowing.

Let r " " " - p & q.

i.e r is the statement. "The Sun is rising in the East. and it is Snowing.

Connectives:-

There are five logic Connectives which are frequently used.

(1) Negation ($\sim$ (or) ($\neg$)

(2) Conjunction ($\wedge$)

(3) Disjunction ($\vee$)

(4) implication ($\rightarrow$) or Conditional

(5) Bi-implication($\leftrightarrow$) or Bi- Conditional.

S.NO	Symbol	Name	Connective word
1	$\sim$ or $\neg$	Negation	Not
2.	$\wedge$	Conjunction	and
3	$\vee$	Disjunction	or
4	$\rightarrow$	implication (or) Conditional	implies (or) If – then –
5.	$\rightleftarrows$ (or) $\leftrightarrow$	Bi- Conditional (or) Equivalence	If and only if

If P and Q are any two statements then

$\neg P$ or $\sim P$, $P \wedge Q$, $P \vee Q$, $P \rightarrow Q$, $P \leftrightarrow Q$ are also statements.

Any statement which do not contain any of the connectives is called as the "atomic statement" or primary or primitive statement.

Eg:- Ravi is taller than Mahesh
4 > 3.

Those statements which contain one or more atomic statements & some connectives are called molecular or composite or compound statements.

Truth Table:- Determine the truth value of a statement formula for each possible combination of the truth values of the compound statement. A table showing ~~called~~ of the compound statement. A table showing all such truth values is called the truth table of the formula.

Negation:- (not, ~)

If P denotes a statement, then the negation of P is written as ~P & is read as "not P".

Truth table for negation

P	¬P or ~P
T	F
F	T

1. Eg:- P - Chennai is a city.
~P - Chennai is not a city.

Eg:- P — I went to my College yesterday
~P : I did not go to my College yesterday.

2. Conjunction :- Conjunction of two statements P & Q is denoted by the statement $P \wedge Q$, which read as P & Q

The truth ~~table~~ value of $P \wedge Q$ is true only if both P & Q are true. otherwise $P \wedge Q$ is false.

The truth table of $P \wedge Q$ is as follows

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Eg:- P :- Today is Monday
Q :- Ther are 50 tables in this room.

$P \wedge Q$:- Today is Monday & there are 50 tables in this room.

Eg:- He opened the book & started to read — Conjunction
Ramu & Siva are Cousins :- not a Conjuction

Eg:- P : $\sqrt{2}$ is an irrational number.
Q : 9 is a Prime number.
R : All triangles are Equilateral.
S : $2+5=7$.

P & S are statements

Q & R are false statements

$P \wedge Q$: is false ∵ P is true & Q is false.

$Q \wedge R$: is false ∵ both Q & R are false.

$R \wedge S$: is false ∵ R is false & S is true

$S \wedge P$ is true ∵ S is true & P is true

Disjunction :- (or V)

Disjunction of two statements P & Q may be denoted by the statement "$P \vee Q$" & read as "P or Q" or "P Joint Q".

The truth value of $P \vee Q$ is true if either P is true (or) Q is true (or) both P & Q are true otherwise $P \vee Q$ is false.

Truth table for disjunction

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Eg:- P : $\sqrt{2}$ is an irrational number

Q : $2+3=5$

R : 8 is a Prime number

S : All triangles are issosceles.

$P \vee Q$ is true ∵ both are True.

$Q \vee R$ is true : Q is true R is false.

$R \vee S$ is false ∵ R is false and S is false.

$S \vee P$ is true ∵ S is false and P is true.

Eg :- 50 or 60 Peacocks were killed in the fire yesterday.

I ~~sat~~ shall either watch the game on televisian or go for a movie.

Conditional Statements ($\rightarrow$).

If P and Q are any two statements, then the statement $P \rightarrow Q$, & read as "if P then Q" is called Conditional statement or implication statements.

$P \rightarrow Q$ is false only when Q is false & P is true

Truth table for Conditional statement ($P \rightarrow Q$)

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Eg :- If I get the book. then I begin to read.

P : I get the book.

Q : I began to read.

Symbolic form is $P \rightarrow Q$.

Construct a truth table for the statement $(P \rightarrow Q) \wedge (Q \rightarrow P)$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Eg :- P – ABCD is a rectangle

Q – $5+9>4$.

$P \rightarrow Q$ If ABCD is a rectangle then $5+9>4$.

Note: In the statement $P \rightarrow Q$, P is called the antecedent or hypothesis & Q is called the consequent or conclusion.

Bi-Conditional statement ($P \Leftrightarrow Q$)

If P & Q are any two statements then the statement $P \Leftrightarrow Q$, & read as "P if & only if Q" is called a bi-conditional statement or bi-implication.

The truth value of $P \Leftrightarrow Q$ is true if both P & Q have (similar) identical truth values, otherwise its truth value is false.

Truth table for a bi-conditional statement ($P \to Q$)

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

$P \Leftrightarrow Q$ is same as $(P \to Q) \wedge (Q \to P)$

read it as if p then q. and if q then p.

Construct the truth table for the $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$\sim(P \wedge Q) \Leftrightarrow \sim P \vee$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Construct the truth tables for

(i) $\sim(P \wedge Q)$ (ii) $\sim P \vee \sim Q$ (iii) $\sim(P \vee Q)$ (iv) $\sim P \wedge \sim Q$.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P) \vee (\sim Q)$	$\sim P \wedge \sim Q$
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	F
F	T	F	T	T	F	T	F
F	F	F	T	T	T	T	T

$P \vee Q$.	$\sim(P \vee Q)$.
T	F
T	F
T	F
F	T

Converse, Contrapositive & Inverse

The statement $Q \to P$ is called the Converse of $P \to Q$.

The statement $\sim Q \to \sim P$ is called the Contrapositive of $P \to Q$.

The statement $\sim P \to \sim Q$ is called the inverse of $P \to Q$.

Example:- what are the inverse, converse & contrapositive of the implication "If today is a holiday, then I will go for a movie".

Let P — Today is a holiday.
Q — I will go for a movie.

Then the given statement can be written as $P \to Q$.

(i) The converse of this implication is $Q \to P$
i.e If I go for a movie then today is a holiday.

(ii) The inverse of this implication is $\sim P \to \sim Q$.
i.e If today is not a holiday, then I will not go for a movie

(iii) The contrapositive of $P \to Q$ is the Proposition (statement) $\sim Q \to \sim P$ i.e "If I donot go for a movie, then today is not a holiday.

Definition: Any statement which do not contain any of the connectives is called atomic statement (or) Primary (or) Primitive statement.

Molecular (Composite or Compound) statements:

Those statements which contain one or more atomic statements & some connectives are called Molecular statements

Eg:- $\neg P$, $P \wedge Q$, $P \wedge \neg Q$.

Write the following statement in symbolic form

1. **Statement**:- If either John prefers tea or Jim prefers coffee then Rita prefers milk.

A : John prefers tea

B : Jim prefers coffee.

C : Rita prefers milk.

Symbolic form is $(A \vee B) \rightarrow C$.

2. **statement**:- If Rita & Sita go to IT camp & Jim and John go to P.C. camp then the College gets the good name.

A: Rita goes to IT camp

B: Sita goes to IT camp

C: Jim " " PC camp

D: John " " PC camp

E: College gets the good name.

Symbolic form. $(A \wedge B) \wedge (C \wedge D) \rightarrow E$

1

3. If the sun is shining today, then $2+3>4$.

A : The Sun is shining today

B : $2+3>4$.

Symbolic form $A \rightarrow B$.

4. The Crop will be destroyed if there is a flood.

A : The Crop will be destroyed.

B : There is a flood.

Symbolic form $B \rightarrow A$.

5. If Either Ramu takes Calculus or Siva takes Sociology, then Venkat will take English

A : Ramu takes Calculus

B : Siva takes Sociology

C : Venkat takes English

Statement : $(A \cup B) \rightarrow C$.

6. P : A circle is a Conic

Q : $\sqrt{5}$ is an irrational number.

R : Exponential Series is Convergent.

Express the following Compound Propositions in words

(i) $P \wedge (\neg Q)$

A circle is a conic and $\sqrt{5}$ is not an irrational number.

(ii) $(\neg P) \vee Q$.

A circle is not a Conic or $\sqrt{5}$ is an irrational number.

(iii) $Q \to \neg P$

If $\sqrt{5}$ is an irrational number then a circle is not a Conic.

(iv) $\neg P \leftrightarrow \{Q \wedge \neg R\}$.

A circle is not a Conic iff $\sqrt{5}$ is an irrational number & the Exponential Series is not Convergent.

(v) If Either John Prefers tea or Jim Prefers Coffee, then Rita Perfers milk.

A : John Prefers tea

B : Jim Prefers Coffee.

C : Rita Prefers milk.

Symbolic form $(A \vee B) \to C$.

Well- formed formula (WFF)

A string of symbols containing statement variables, connectives & Parenthesis (brackets) is a WFF if it follows the rules given below.

Rule 1 :- Any statement variable is a WFF

Rule 2 :- If P is a WFF, then $\sim P$ is also a WFF

Rule 3 :- If P & Q are WFF, then $(P \wedge Q)$, $(P \vee Q)$, $(P \rightarrow Q)$ $(P \Leftrightarrow Q)$ are also WFF

Eg :- $\sim(P \wedge Q)$, $\sim(P \vee Q)$, $(P \rightarrow (P \vee Q))$, $(P \rightarrow (Q \rightarrow R))$ are WFF.

1) $\sim P \wedge Q$ is not well formed formula because a WFF would be either $(\neg P \wedge Q)$ or $\sim(P \wedge Q)$. Here $\sim P \wedge Q$ does not contains Parenthesis.

2) $(P \rightarrow Q) \rightarrow (\wedge Q)$ is not WFF because $\wedge Q$ is not statement variable.

3) $(P \rightarrow Q$ is not WFF because one of the Parenthesis is missing ie $P \rightarrow Q$.

4) $((P \wedge Q) \rightarrow Q)$ is a WFF but $(P \wedge Q) \rightarrow Q$ is not a WFF

From the formulas given below select those which are well formed according to definition.

(i) $(P \to (P \vee Q))$ is a wff

(ii) $((P \to (\sim P)) \to (\sim P))$ is a wff

(iii) $((P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ it is not wff because one of the Parenthesis in the begining is missing.

(iv) $((\sim P \to Q) \to (Q \to R)))$ it is not wff because one more Parenthesis in the End.

(v) $((P \wedge Q) \Leftrightarrow P)$ it is a wff.

Note: For "n" simple Propositions, the total number of rows will be 2^n.

Construct the truth tables for the following.

(a) $[(P \vee Q) \wedge (\sim R)] \Leftrightarrow Q$.

P	Q	R	$\sim R$	$P \vee Q$	$(P \vee Q) \wedge (\sim R)$	$(P \vee Q) \wedge (\sim R) \Leftrightarrow Q$
T	T	T	F	T	F	F
T	T	F	T	T	T	T
T	F	T	F	T	F	T
F	T	T	F	T	F	F
T	F	F	T	T	T	F
F	T	F	T	T	T	T
F	F	T	F	F	F	T
F	F	F	T	F	F	T

(b) Construct the truth table for $P \wedge \sim P$

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

(c) $\sim (\sim P \vee \sim Q)$

P	Q	$\sim P$	$\sim Q$	$(\sim P \vee \sim Q)$	$\sim (\sim P \vee \sim Q)$
T	T	F	F	F	T
F	T	T	F	T	F
T	F	F	T	T	F
F	F	T	T	T	F

(d) $P \wedge (P \vee Q)$

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$
T	T	T	T
F	T	T	F
T	F	T	T
F	F	F	F

(e) $\sim P \wedge (Q \wedge P)$.

P	Q	$\sim P$	$Q \cap P$	$\sim P \wedge (Q \wedge P)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

(f) $(P \rightarrow Q) \Leftrightarrow (\sim Q \rightarrow \sim P)$

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$	$\sim Q \rightarrow \sim P$	$(P \rightarrow Q) \Leftrightarrow (\sim Q \rightarrow \sim P)$
T	T	F	F	T	T	
T	F	F	T	F	T	
F	T	T	F	T	F	
F	F	T	T	T	T	

j) $(P \to Q) \Leftrightarrow (\sim Q \to \sim P)$

P	Q	$\sim P$	$\sim Q$	$P \to Q$	$Q \to P$ (Converse)	$\sim P \to \sim Q$	$\sim Q \to \sim P$	$(P\to Q) \Leftrightarrow (\sim P \vee Q)$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	T	F	T
F	T	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T	T

All the entries in the last Column are T

$\therefore$ $(P \to Q) \Leftrightarrow (\sim Q \to \sim P)$ is a tautology.

Examples

Let P be "He is tall."

Q: He is handsome. write each of the following in symbolic form using P & Q.

(a) He is tall & handsome - $P \wedge Q$

(b) He is tall but not handsome: $P \wedge \sim Q$

(c) He is neither tall nor handsome: $\sim P \wedge \sim Q$

Example: Translate the following Sentences into Propositional forms & Symbolic form.

Let P: A man is rich

Q: A man is happy.

(a) A man is rich & happy.

$P \wedge Q$

(b) A man is rich but not happy

$P \wedge \sim Q$

(c) A man is rich or happy

$P \vee Q$

(d) A man is neither rich nor happy.

$\sim P \wedge \sim Q$

<u>Example</u>. Let P: Ravi is rich and Q: Ravi is happy.

write each of the following in symbolic form.

(i) Ravi is poor but happy

$\sim P \wedge Q$

(ii) Ravi is neither rich nor happy

$\sim P \wedge \sim Q$

(iii) Ravi is rich & unhappy

$P \wedge \sim Q$

Tautology :- A Prepositional function or statement is said to be a tautology if it is true always

Eg :- $p \vee \sim p$ is a tautology

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Contradiction :- A Prepositional function or statement is said to be a Contradiction if it is always false.

Eg: $p \wedge \sim p$ is a Contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Note (i) $p \vee \sim p$ is a tautology $\because$ it always has truth value T
(ii) $p \cap \sim p$ is a contradiction since it always has true value F.

Contigency :- A Prepositional function which is neither tautology nor a Contradiction is Called a Contigency.

Logically Equivalent :-

The Prepositions A and B are said to be logically Equivalent if they have identical truth values or $A \Leftrightarrow B$ is a tautology. The symbol $\Leftrightarrow$ stands for logically Equivalent to and is written as $A \Leftrightarrow B$ or $A = B$.

Substitution instance:

A formula A is called a substitution instance of another formula B if A can be obtained from B by substituting formulae for some variables of B, with the condition that the same formula is substituted for the same variable each time it occurs.

Ex:- Let $A: P \rightarrow (Q \wedge P)$

Substitutes $R \Leftrightarrow S$ for P in A, we get

$$B: (R \Leftrightarrow S) \rightarrow (Q \wedge (R \Leftrightarrow S))$$

Then B is said to be substitute instance of A.

Two formulas A & B are said to be equivalent to each other if and only if $A \Leftrightarrow B$ is a tautology.

The equivalence of two formulas A & B is denoted by $A \Leftrightarrow B$, which is read as A is Equivalent to B.

Note (i) $\Leftrightarrow$ is only symbol, but not connective.

(ii) $A \Leftrightarrow$ if and only if the truth values of A, B are same.

i.e the final columns in the truth tables of A, B are same.

i.e the truth tables of A, B are same.

(iii) Equivalence relation is symmetric & transitive.

Ex:- (i) $\sim(\sim P)$ is Eaivalent to P.

(ii) $P \wedge P$ is " " P.

(iii) $P \vee \sim P$ is " " $Q \vee \sim Q$

(iv) $(P \wedge \sim P) \vee Q$ is " " Q.

Equivalent formulas

Let P, Q, R be any 3 statements. Then all Possible formulas may be written as

(i) $P \vee P \Leftrightarrow P$; $P \wedge P \Leftrightarrow P$; idemPotent law.

(2) $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ ⟶ Associative Laws

(3) $P \vee Q \Leftrightarrow Q \vee P$
$P \wedge Q \Leftrightarrow Q \wedge P$ ⟶ Commutative laws.

(4) $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ ⟶ Distributive laws.

(5) $P \wedge T \Leftrightarrow P$
$P \vee F \Leftrightarrow P$ ⟶ Identity ~~domination~~ Laws.

(6) $P \vee T \Leftrightarrow T$
$P \vee F \Leftrightarrow F$ ⟶ domination ~~double negation~~ law

7) $\sim(\sim P) \Leftrightarrow P$ ⟶ double negation law

8) $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$ $\longrightarrow$ Demorgan's Law

$\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

9) $P \vee (P \wedge Q) \Leftrightarrow P$ $\longrightarrow$ absorption laws

$P \wedge (P \vee Q) \Leftrightarrow P$

10) $P \vee \sim P \Leftrightarrow T$ $\longrightarrow$ negation laws.

$P \wedge \sim P \Leftrightarrow P$

*Problem :- write the statement in symbolic form and find whether it is a tautology.

Problem: If I am hungry and thirsty then I am hungry

P : I am hungry

Q : I am thirsty

$(P \wedge Q) \longrightarrow P$

P	Q	$P \wedge Q$	$(P \wedge Q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Problem: Prove that $P \wedge (Q \vee R) \Leftrightarrow ((P \wedge Q) \vee (P \wedge R))$

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	T	F	T	F	F	F	T	T
T	T	F	T	T	T	F	F	T
F	F	F	F	F	F	F	F	T

Problem: Prove that $(P \wedge Q) \vee R \Leftrightarrow ((\sim P \vee \sim Q) \wedge \sim R)$ is a contradiction.

Sol

~~Q~~ P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$	$\sim P \vee \sim Q$	$(\sim P \vee \sim Q) \wedge \sim R$	$(P \wedge Q) \vee R \Leftrightarrow ((\sim P \vee \sim Q) \wedge \sim R)$
T	T	T	T	T	F	F	F
T	F	F	F	F	T	T	F
T	T	F	T	T	F	F	F
F	T	T	F	T	T	F	F
F	F	T	F	T	T	F	F
T	F	T	F	T	T	F	F
F	T	F	F	F	T	T	F
F	F	F	F	F	T	T	F

Problem :- show that $P \rightarrow (Q \rightarrow P)$ and $\sim P \rightarrow (P \rightarrow Q)$ are logically Equivalent.

Sol Let $A = P \rightarrow (Q \rightarrow P)$

$B = \sim P \rightarrow (P \rightarrow Q)$

A

P	Q	$Q \rightarrow P$	$P \rightarrow (Q \rightarrow P)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

B

$P \rightarrow Q$	$\sim P$	$\sim P \rightarrow (P \rightarrow Q)$
T	F	T
F	F	T
T	T	T
T	T	T

$\therefore A \equiv B$

$\therefore P \rightarrow (Q \rightarrow P) = \sim P \rightarrow (P \rightarrow Q)$

Problem

Show the following Equivalents $P \to (Q \to P) \Leftrightarrow \sim P \to (P \to Q)$

Sol L.H.S $P \to (Q \to P)$

$\Rightarrow P \to (\sim Q \vee P)$ } implication law

$\Rightarrow \sim P \vee (\sim Q \vee P)$

$\Rightarrow \sim P \vee (P \vee \sim Q)$ (Associative law)

$\Rightarrow (\sim P \vee P) \vee \sim Q$ (Commutative law)

$\Rightarrow T \vee \sim Q$ ($\sim P \vee P = T$ negative law)

$= T$

R.H.S. $\sim P \to (P \to Q)$

$\Rightarrow \sim P \to (\sim P \vee Q)$ } implication law

$\Rightarrow \sim(\sim P) \vee (\sim P \vee Q)$

$\Rightarrow P \vee (\sim P \vee Q)$ (double negation law)

$\Rightarrow (P \vee \sim P) \vee Q$

$\Rightarrow T \vee Q$

$\Rightarrow = T$

LHS = R.H.S.

Problem . $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

L.H.S $(P \to Q) \wedge (R \to Q)$

$\Leftrightarrow (\sim P \vee Q) \wedge (\sim R \vee Q)$ (implication law)

$\Leftrightarrow (\sim P \wedge \sim R) \vee Q$ (Distributive law)

$\Leftrightarrow \sim (P \vee R) \vee Q$ $[\because \sim (P \vee R) \Leftrightarrow \sim P \wedge \sim R]$

$\Leftrightarrow (P \vee R) \to Q$ (implication law)

Problems:- If Siva likes basket ball then he likes swimming and gymnastics.

Question. Siva dislike basket ball or he likes both Swimming and gymnastics.

Find whether these two statements are Equivalent.

Sol P: Siva likes basket ball

Q: He likes swimming

R: he likes gymnastics

$P \to (Q \wedge R)$

P: Siva dislike basket ball

Q: Siva likes Swimming

R: Siva likes gymnastics

$\sim P \vee (Q \wedge R)$

P	Q	R	$\sim P$	$Q \wedge R$	$P \to (Q \wedge R)$ A	$\sim P (Q \wedge R)$ B.	$A \Leftrightarrow B$
T	T	T	F	T	T	T	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
T	T	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

Prob Are $(P \to Q) \to R$ and $P \to (Q \to R)$ are logically Equivalent. Justify your answer by using rules of logic to simplify both formulas and also by using truth table.

Sol $(P \to Q) \to R$

$\sim (P \to Q) \vee R$

$\sim (\sim P \vee Q) \vee R$

$(P \wedge \sim Q) \vee R$

$P \to (Q \to R) \Rightarrow P \to (\sim Q \vee R)$

$\sim P \to (\sim Q \vee R)$

$(\sim P \vee \sim Q) \vee R$

P	Q	R	$P \to Q$	$(P \to Q) \to R$	$Q \to R$	$(P \to (Q \to R))$
T	T	T	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
T	T	F	T	F	F	F
F	T	F	T	F	F	T
F	F	T	T (doubt)	T	T	T
F	F	F	T	F	T	T
F	T	T	T	T	T	T

The two statements are not logically Equivalent.

Problem. $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$

L.H.S $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$\Rightarrow ((\sim P \wedge \sim Q) \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$ (Associative law)

$\Rightarrow (\sim (P \vee Q) \wedge R) \vee ((Q \vee P) \wedge R)$ (Demorgan's and distributive law)

$\Rightarrow (\sim (P \vee Q) \wedge R) \vee ((P \vee Q) \wedge R)$ (commutative)

$\Rightarrow (\sim (P \vee Q) \vee (P \vee Q)) \wedge R$ (Distributive)

$\Rightarrow T \wedge R$ $(\because \sim P \vee P = T)$ (identity)

$\Rightarrow R$

Problem $\sim(P\wedge Q) \rightarrow (\sim P \vee \sim(P\vee Q)) \Leftrightarrow \sim P \vee Q$.

$\Rightarrow \sim(\underbrace{P\wedge Q}_{P}) \rightarrow (\sim P \vee \sim(P\vee Q))$ $[P \rightarrow Q = \sim P \vee Q]$

$\Rightarrow \sim(\sim(P\wedge Q)) \vee (\sim P \vee \sim(P\vee Q))$ [implication law)

$\Rightarrow (P\wedge Q) \vee (\sim P \vee \sim(P\vee Q))$. [double negation]

$\Rightarrow (P\wedge Q) \vee [\sim P \vee \sim P] \vee Q]$ [Associative]

$\Rightarrow (P\wedge Q) \vee [\sim P \vee Q]$

$\Rightarrow (P\vee \sim P) \wedge (Q \vee \sim P) \vee Q$ (distributive]

$\Rightarrow T \wedge (Q\vee \sim P) \vee Q$ (negation)

$\Rightarrow (Q \vee \sim P) \vee Q$ $(T \cap P = P)$

$\Rightarrow (Q\vee Q) \vee \sim P$ (Associative)

$\Rightarrow Q \vee \sim P$

$\sim P \vee Q$ (Commutative)

Problem

$(P\vee Q)\wedge\sim(\sim P\wedge(\sim Q\vee\sim R))\vee((\sim P\wedge\sim Q)\vee(\sim P\wedge\sim R))$

$\Rightarrow (P\vee Q)\wedge(P\vee\sim(\sim Q\vee\sim R))\vee((\sim P\wedge\sim Q)\vee(\sim P\wedge\sim R))$

$((P\vee Q)\wedge(P\vee(Q\wedge R))\vee((\sim P\vee\sim P)\wedge(\sim Q\vee\sim R))$

$((P\vee Q)\wedge R)\vee(\sim P\wedge\sim(Q\wedge R)]$

$((P\vee Q)\wedge R]\vee\sim(P\vee(Q\wedge R)$

$((P\vee(Q\wedge R))\vee\sim(P\vee(Q\wedge R)))$ $[P\vee P=T]$

$=T$

Problem show that $\sim(P\Leftrightarrow Q)\Leftrightarrow(P\vee Q)\wedge\sim(P\wedge Q)$

$\rightarrow \sim(P\Leftrightarrow Q)$

$\Rightarrow \sim((P\rightarrow Q)\wedge(Q\rightarrow P))$ $(P\Leftrightarrow Q=(P\rightarrow Q)\wedge(Q\rightarrow P))$

$\Rightarrow \sim((\sim P\vee Q)\wedge(\sim Q\vee P))$ (implication)

$\Rightarrow \sim(\sim P\vee Q)\vee\sim(\sim Q\vee P)$ (Demorgan's)

$\Rightarrow (P\wedge\sim Q)\vee(Q\wedge\sim P)$ (double negation)

$\Rightarrow (P\vee Q)\wedge\sim(Q\wedge P)$

$\Rightarrow (P\vee Q)\wedge\sim(P\wedge Q)$.

Functionally Complete sets of connectives :- So, far we have the Connection $\wedge, \vee, \sim P, \rightarrow$ and $\rightleftarrows$. Now we introduce some other Connectives namely NAND and NOR which have useful applications in the design of Computers. The word "NAND" is a Combination of "NOT" and "AND" while the word "NOR" is a Combination of ~~NO~~ "NOT" and "OR" where NOT ~~satisf~~ stands for negation AND stands for Conjuction and OR stands for disjunction.

The connective NAND is denoted by the Symbol ~~A~~. $\uparrow$ and its is defined as $P \uparrow Q \Leftrightarrow \sim(P \wedge Q)$, for any two formulas P and Q.

The Connective NOR is denoted by the Symbol $\downarrow$ and it is defined as $P \downarrow Q \Leftrightarrow \sim(P \vee Q)$, for any two formulas P and Q.

Note Connective $\uparrow$ and $\downarrow$ are duals of each other.

P	Q	$P \wedge Q$	NAND $= P \uparrow Q$ $\sim(P \wedge Q)$	$P \vee Q$	NOR $= P \downarrow Q$ $\sim(P \vee Q$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	T	F	T

Exclusive disjunction
suppose P, Q are any two formulas and the Connective Exclusive OR "or" Exclusive disjunction is denoted by the symbol $\bar{\vee}$ and truth value of $P \bar{\vee} Q$ is true whenever either P or Q is true. but not both true.

P	Q	$P \bar{\vee} Q$.
T	T	F
T	F	T
F	T	T
F	F	F

(i) $\sim P \Leftrightarrow \sim P \wedge \sim P$
$\Leftrightarrow \sim(P \vee P)$
$\Leftrightarrow P \downarrow P$

$\sim P \Leftrightarrow \sim P \vee \sim P$
$\Leftrightarrow \sim(P \wedge P)$
$\Leftrightarrow P \uparrow P$.

The following equivalences follow from the definition of connective $\bar{\vee}$.

① $P \bar{\vee} Q \Leftrightarrow Q \bar{\vee} P$

② $(P \bar{\vee} Q) \bar{\vee} R \Leftrightarrow P \bar{\vee} (Q \bar{\vee} R)$

③ $P \wedge (Q \bar{\vee} R) \Leftrightarrow (P \wedge Q) \bar{\vee} (P \wedge R)$

④ $(P \bar{\vee} Q) \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge Q)$

⑤ $(P \bar{\vee} Q) \Leftrightarrow \sim (P \rightleftarrows Q)$

Functionally complete :- A set of connectives is said to be functionally complete connective if every formula can be expressed interms of an equivalent formula containing the connectives only from this set.

we have

① $P \vee Q \Leftrightarrow \sim(\sim P \wedge \sim Q)$

② $P \wedge Q \Leftrightarrow \sim(\sim P \vee \sim Q)$

③ $P \rightarrow Q \Leftrightarrow \sim P \vee Q$

④ $P \rightleftarrows Q \Leftrightarrow (\sim P \vee Q) \cap (P \vee \sim Q)$

the set of connectives $\{\wedge, \sim\}$ and $\{\vee, \sim\}$ are functionally complete set.

the sets $\{\sim\}$, $\{\wedge\}$, $\{\vee\}$, $\{\wedge, \vee\}$ are not functionally complete set.

1) write an Equivalent formula for $P \wedge (Q \rightleftarrows R) \vee (R \rightleftarrows P)$ which does not Contain the Bi-Conditional.

$$P \wedge (Q \rightleftarrows R) \vee (R \rightleftarrows P) \Leftrightarrow P \wedge ((Q \to R) \wedge (R \to Q)) \vee ((R \to P) \wedge (P \to R))$$

(2) write an Equivalent formula for $P \wedge (Q \rightleftarrows R)$ which Contains neither the Bi-Conditional nor the Conditional.

$$P \wedge (Q \rightleftarrows R) \Leftrightarrow P \wedge ((Q \to R) \wedge (R \to Q))$$

$$\Leftrightarrow P \wedge ((\sim Q \vee R) \wedge (\sim R \vee Q))$$

Some Properties of the Connectives NAND and NOR

1) $P \uparrow Q \Leftrightarrow Q \uparrow P$; $P \downarrow Q \Leftrightarrow Q \downarrow P$ (Commutative)

(2) The connectives $\uparrow$ and $\downarrow$ are not associative

Because $P \uparrow (Q \uparrow R) \Leftrightarrow P \uparrow \sim (Q \wedge R) \Leftrightarrow \sim (P \wedge \sim (Q \cap R))$

$$\Leftrightarrow \sim P \vee (Q \cap R)$$

where $(P \uparrow Q) \uparrow R \Leftrightarrow \cancel{(P \wedge Q) \vee \sim R}\ \sim (P \wedge Q) \uparrow R$

$$\Leftrightarrow \sim (\sim (P \cap Q) \wedge R)$$

$$\Leftrightarrow (P \wedge Q) \vee \sim R.$$

lly $\downarrow$ is not associative

(3) $P \uparrow Q \uparrow R \Leftrightarrow \sim (P \wedge Q \wedge R)$

(4) $P \uparrow Q \Leftrightarrow \sim(P \wedge Q)$

$\Leftrightarrow \sim P \vee \sim Q$

$\Leftrightarrow (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$.

$P \downarrow Q \Leftrightarrow \sim(P \vee Q)$

$\Leftrightarrow \sim P \wedge \sim Q$

$\Leftrightarrow (\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

Problem

Show that $\{\vee, \sim\}$ is functionally complete set.

It is enough to show that $\sim$ and $\vee$ can be expressed interms of NOR alone then the set $\{\downarrow\}$ is functionally complete set.

Eg: $\sim P \Leftrightarrow \sim P \wedge \sim P$

$\Leftrightarrow \sim(P \vee P)$

$\Leftrightarrow P \downarrow P$.

Normal Forms

A statement formula is said to be in normal form (or) Canonical form if

(1) only the three connectives $(\sim, \wedge, \vee)$ have been used

(2) Distributive laws has been applied

(3) $\sim$ has not been used for a group of letters

(4) Parenthesis has not been used for the same Connective.

NORMAL FORMS (Canonical form)

We denote the word "Product" in place of "Conjunction" and "sum" in place of "disjunction".

A Product of the variables and their negations in a formula is called an elementary Product

Example :- Suppose P and Q are any atomic variables, then P, $\sim P$, $\sim P \wedge Q$, $\sim Q \wedge P \wedge \sim P$, $P \wedge \sim P$ etc are some of elementary Products.

A sum of the variables and their negations in a formula is called an elementary sum

Example :- ~~Q, Q∨∼P, ∼P∨∼Q are some factors of Q∨∼P∨∼Q.~~

Example P, $\sim P$, $\sim P \vee Q$, $\sim P \vee Q \vee \sim P$ etc. are some elementary sums.

Any Part of an elementary Product (sum) which is itself an elementary Product (sum) is called a factor of the original elementary Product (sum).

Example :- Q, $Q \vee \sim P$, $\sim P \vee \sim Q$ are some factor of $Q \vee \sim P \vee \sim Q$

Example Consider

$P \wedge \sim P$ is identically false which is a factor of $P \wedge \sim Q \wedge \sim P$.

Hence $P \wedge \sim Q \wedge \sim P$ is also identically false.

Hence, we Can write that

Result (1) A necessary and sufficient Condition for an elementary Product to be identically false is that it Contains atleast one Pair of factors in which one is the negation of the other.

I/y we Can write

Result (2). A necessary and sufficient Condition for an elementary sum to be identically true is that it Contains atleast one Pair of factors in which one is the negation of the other

Example $P \vee \sim P$ is a factor of $P \vee \sim Q \vee \sim P$ which is identically true.

Disjunction normal form (d.n.f) Suppose A and B are formulas which are Equivalent to B Contains a sum of Elementary Products. Then B is Called a disjunction normal form of A.

(or)

A formula which is Equivalent to a given formula and which Contains a sum of elementary Products is Called a disjunctive normal form of the given formula.

Procedure to obtain a disjunction normal form of a given formula

1) obtain an Equivalent formula by replacing "$\rightarrow$" and "$\rightleftarrows$" with $\wedge$, $\vee$ and $\sim$.

Example :- $P \rightarrow Q$ Can be replaced by $\sim P \vee Q$ and $P \rightleftarrows Q$ (*)
~~$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$~~
Can be replaced by either $\Rightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$ or $(\sim P \vee Q) \wedge (\sim Q \vee \sim P)$

$$\sim P \wedge (\sim Q \vee P) \vee (Q \wedge (\sim Q \vee P))$$

$$(\sim P \wedge \sim Q) \vee (\underbrace{\sim P \wedge P}_{T}) \vee (\underbrace{Q \wedge \sim Q}_{F}) \vee (Q \wedge P)$$

$$(\sim P \wedge \sim Q) \vee (Q \wedge P)$$

2) If the negation is applied to the formula "or" a part of the formula and not to the variables in it, then by using Demorgan in laws an Equivalent formula Can be obtained in which the negation is applied to the variables only.

3) Apply the distribution law until we get a sum of elementary Product is obtained and apply idempotent law if necessary. Thus, we get disjunction normal form of a given formula.

(*). $P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$
$= (P \wedge Q) \vee (\sim P \wedge \sim Q)$

Note 1. The d.n.f of a given formula is not unique. because different d.n.f can be obtained for a given formula if the distribution laws are applied in different ways.

Note: A given formula is identically false if every elementary product appearing in its d.n.f is identically false.

Problem

obtain d.n.f of the following formulas.

(or) find the disjunctive normal form of the following formulae

1) $P \wedge (P \to Q) = P \wedge (\sim P \vee Q)$

$= (P \wedge \sim P) \vee (P \wedge Q).$

2) $Q \to (Q \to P) = Q \to (\sim Q \vee P)$

$= \sim Q \vee (\sim Q \vee P)$

$= (\sim Q \vee \sim Q) \vee P$

$= \sim Q \vee P$

3) $\sim (P \vee Q) \rightleftarrows (P \wedge Q) \Leftrightarrow \underbrace{\sim (P \vee Q)}_{(\sim P \wedge \sim Q)} \wedge (P \wedge Q)) \vee ((P \vee Q) \wedge \sim (P \wedge Q))$

$\Rightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee (P \vee Q) \wedge (\sim P \vee \sim Q)$

$\Rightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee (P \wedge (\sim P \vee \sim Q) \vee (Q \wedge (\sim P \vee \sim Q))$

$\Rightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee (P \wedge \sim P) \vee (P \wedge \sim Q) \vee (Q \wedge \sim P) \vee (Q \wedge \sim Q)$

which is in d.n.f

4) $(Q \to P) \wedge (\sim P \wedge Q) = (\sim Q \vee P) \wedge (\sim P \wedge Q)$

$= (\sim Q \wedge \sim P \wedge Q) \vee (P \wedge \sim P \wedge Q)$

which is in d.n.f

⑤ obtain d.n.f of $P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$\Rightarrow \sim P \vee ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$\Rightarrow \sim P \vee ((\sim P \vee Q) \wedge \sim(\sim(Q \wedge P)))$

$\Rightarrow \sim P \vee ((\sim P \vee Q) \wedge (Q \wedge P))$

$\cancel{\Rightarrow \sim P \vee ((\sim P \vee Q)}$

$\sim P \vee ((\sim P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P))$

$\sim P \vee (\sim P \wedge P \wedge Q) \vee (P \wedge Q)$.

Conjunctive normal form

A formula which is Equivalent to a given formula and which consists of a product of elementary sum is called conjunctive normal form of the given formula.

1) $P \wedge (P \to Q) = P \wedge (\sim P \vee Q)$

2) $((P \to Q) \wedge \sim Q) \to \sim P$

$((\sim P \vee Q) \wedge \sim Q) \to \sim P$

$\sim((\sim P \vee Q) \wedge \sim Q) \vee \sim P$

$\sim((\sim P \vee Q) \vee \sim(\sim Q)) \vee \sim P$

$((P \wedge \sim Q) \vee Q) \vee \sim P \equiv ((P \vee Q) \wedge (\sim Q \vee Q)) \vee \sim P$

$(P \vee Q \vee \sim P) \wedge (\sim Q \vee Q \vee \sim P)$

(iii) $((P \to Q) \wedge \sim P) \to \sim Q$

$((\sim P \vee Q) \wedge \sim P) \to \sim Q$

$\sim((\sim P \vee Q) \wedge \sim P) \vee \sim Q$

$\sim(\sim P \vee Q) \vee \sim(\sim P) \vee \sim Q$

$((P \wedge \sim Q) \vee P) \vee \sim Q$

$((P \wedge \sim Q) \vee P) \vee \sim Q$

$((P \vee P) \wedge (\sim Q \vee P)) \vee \sim Q$

$(P \vee P \vee \sim Q) \wedge (\sim Q \vee P \vee \sim Q)$

$(P \vee \sim Q) \wedge (\sim Q \vee P)$

(iv) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

$(\sim(P \vee Q) \to (P \wedge Q)) \wedge ((P \wedge Q) \to \sim(P \vee Q))$

$((P \vee Q) \vee (P \wedge Q)) \wedge (\sim(P \wedge Q) \vee \sim(P \vee Q))$

$((P \vee Q) \vee (P \wedge Q)) \wedge ((\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q))$

$((P \vee Q \vee P) \wedge (P \vee Q \vee Q)) \wedge ((\sim P \vee \sim Q \vee \sim P) \wedge (\sim Q \vee \sim Q \vee \sim P))$

Min terms :-

Min terms consists of conjunction of variables or its negation but not both appears only once.

for the variables P, Q there are 2^2 min terms

Example :- $(P \wedge Q)$; $(P \wedge \sim Q)$; $(\sim P \wedge Q)$; $(\sim P \wedge \sim Q)$

for the variables P, Q and R there are 2^3 min terms.

Example :- $(P \wedge Q \wedge R)$; $(P \wedge Q \wedge \sim R)$; $(\sim P \wedge Q \wedge R)$; $(P \wedge \sim Q \wedge R)$

$(\sim P \wedge \sim Q \wedge \sim R)$; $(P \wedge \sim Q \wedge \sim R)$; $(\sim P \wedge \sim Q \wedge R)$; $(\sim P \wedge Q \wedge \sim R)$

Truth table for min terms :-

P	Q	~P	~Q	P∧Q	P∧~Q	~P∧Q	~P∧~Q
T	T	F	F	T	F	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	F	T	F
F	F	T	T	F	F	F	T

Note : No two min terms are equivalent.

Note :- The number of min terms applying in the normal form is the same as the number of entries with the true value T in the truth table. They even formula has an equivalent P.d.n.f such a normal form is unique.

Max terms

It consists of disjunction of variables or its negation but not both appears only once.

For the variables P and Q there are 2^2 max terms

Example $(P \vee Q)$; $(\sim P \vee Q)$; $(P \vee \sim Q)$; $(\sim P \vee \sim Q)$

For the variables P, Q and R there are 2^3 max terms.

Example: $(P \vee Q \vee R)$, $(\sim P \vee Q \vee R)$; $(P \vee \sim Q \vee R)$; $(P \vee Q \vee \sim R)$;
$(\sim P \vee \sim Q \vee R)$; $(P \vee \sim Q \vee \sim R)$; $(\sim P \vee Q \vee \sim R)$; $(\sim P \vee \sim Q \vee \sim R)$

Truth table for Max terms

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim P \vee Q$	$P \vee \sim Q$	$\sim P \vee \sim Q$
T	T	F	F	T	T	T	F
T	F	F	T	T	F	T	T
F	T	T	F	T	T	F	T
F	F	T	T	F	T	T	T

Note :- No two Max terms are equivalent.

Principle disjunctive normal form (sum of Products Canonical form)

An Equivalent formula consisting of disjunction of min terms only is known as Principle disjunctive normal form. There are two methods to obtain Principle disjunctive normal formula of the given formula.

1) Truth table method
2) Replacement method.

Problems

1) Find P.d.n.f of the following.

(i) $P \rightarrow Q$.

→ Min terms: $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$P \wedge \sim Q$	$\sim P \wedge Q$	$\sim P \wedge \sim Q$	$P \rightarrow Q$
T	T	F	F	T	F	F	F	T
T	F	F	T	F	T	F	F	F
F	T	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T	T

$$P.d.n.f = (P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$$

(True వచ్చినవి మాత్రమే తీసుకోవాలి)

Note: $P \wedge Q = T$

(ii) $P \vee (\sim P \rightarrow (Q \vee (\sim Q \rightarrow R)))$

Note. $\sim P \rightarrow (Q \vee (\sim Q \rightarrow R)) = A$

P	Q	R	$\sim P$	$\sim Q$	$\sim Q \rightarrow R$	$Q \vee (\sim Q \rightarrow R)$	$\underbrace{\sim P \rightarrow (Q \vee (\sim Q \rightarrow R))}_{A}$	$P \vee A$
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	T	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	F	F

Min Terms

$P \wedge Q \wedge R$

$P \wedge Q \wedge \sim R$

$P \wedge \sim Q \wedge R$

$P \wedge \sim Q \wedge \sim R$

$\sim P \wedge Q \wedge R$

$\sim P \wedge Q \wedge \sim R$

$\sim P \wedge \sim Q \wedge R$

$\sim P \wedge \sim Q \wedge \sim R$.

P.d.n.f (A) : $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R)$

$\vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

$\sim A = \sim P \wedge \sim Q \wedge \sim R.$

$\sim(\sim A) = P \vee Q \vee R$ = P.c.n.f (Principal Conjunctive formula)

Prob. obtain the ~~Principal dissjunctive normal form of~~

$\sim P \vee Q.$

(o) $\cancel{\sim(P \vee Q)}$.

Replacement Method (without using truth table)

Note 1) Replace Conditional & Bi-Conditional by their Equivalent formula Containing only $\sim, \wedge,$ and $\vee$.

2) use demorgan's laws & distributive Laws.

3) Any Elementary Product which is a Contradiction Can be ignored

(4) If identical min terms appears in the disjunction then the repeated one is ignored.

Note. $P \vee F = P$

$P \wedge F = F$

$P \wedge T = P$

$P \wedge \sim P = F$

$P \vee \sim P = T$

1) Find P.d.n.f for following formulas without using truth table.

(i) $P \Leftrightarrow Q$.

$\rightarrow$ $(P \rightarrow Q) \wedge (Q \rightarrow P$

$(\sim P \vee Q) \wedge (\sim Q \vee P)$ [$\because A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$]

$(\sim P \wedge \sim Q) \vee (\underbrace{\sim P \wedge P}_{F}) \vee (\underbrace{Q \wedge \sim Q}_{F}) \vee (Q \cap P)$

$(\sim P \wedge \sim Q) \vee (Q \cap P)$ is the P.d.n.f.

(ii) $\sim P \vee Q$

$(\sim P \wedge T) \vee (T \cap Q)$

$(\sim P \wedge (\underbrace{Q \vee \sim Q}_{T})) \vee ((\underbrace{P \vee \sim P}_{T}) \wedge Q)$ [$T = Q \vee \sim Q$. $T = P \vee \sim P$.]

$((\sim P \wedge Q) \vee (\sim P \wedge \sim Q)) \vee ((P \wedge Q) \vee (\sim P \wedge Q))$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

(iii) $(P\wedge Q)\vee(\sim P\wedge R)\vee(Q\wedge R)$

$((P\wedge Q)\wedge T)\vee((\sim P\wedge R)\wedge T)\vee((Q\wedge R)\wedge T)$

$((P\wedge Q)\wedge(R\vee\sim R))\vee((\sim P\wedge R)\wedge(Q\vee\sim Q))\vee((Q\wedge R)\wedge(P\vee\sim P))$

$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge R\wedge Q)\vee(\sim P\wedge R\wedge\sim Q)$
$\vee(Q\wedge R\wedge P)\vee(Q\wedge R\wedge\sim P)$

$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge R\wedge Q)\vee(\sim P\wedge R\wedge\sim Q)$
is the p.d.n.f.

(iv) $P\to((P\to Q)\wedge\sim(\sim Q\vee\sim P))$

$\sim P\vee((\sim P\vee Q)\wedge\sim(\sim Q\vee\sim P))$ (implication law)

$\sim P\vee((\sim P\vee Q)\wedge(Q\wedge P))$ (Demorgan's law)

$\sim P\vee((\sim P\wedge(Q\wedge P))\vee(Q\wedge(Q\wedge P))$ (distributive law)

$\sim P\vee((\sim P\wedge P)\wedge Q\vee(Q\wedge Q)\wedge P)$ (Associative law)

$\sim P\vee((F\wedge Q)\vee(Q\wedge P))$

$\sim P\vee(F\vee(Q\wedge P))$

$\sim P\vee(Q\wedge P)$

$(\sim P\wedge T)\vee(Q\wedge P)$

$\sim P\wedge(Q\vee\sim Q)\vee(Q\wedge P)$

$(\sim P\wedge Q)\vee(\sim P\wedge\sim Q)\vee(Q\wedge P)$ is the ~~pdnf~~ p.d.n.f.